Indian Journal of Multidisciplinary Research (IJOMR)
(Yearly, Blind Peer-Reviewed, Multidisciplinary Journal)
Home Page: https://ijomr.org

ISSN: 3048-8036 (Online)
July, 2025. Vol. 2, Issue 1
Page No: 53-67

Mathematical Analysis of Two-phase Blood Flow through a Stenosed Artery with a Permeable Wall

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DOI: https://doi.org/10.70798/IJOMR/020040006
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Received
29/05/2025

Accepted
10/06/2025

Published
09/07/2025

Abstract

This analytical study examines the womersley flow of blood in stenosed arteries, utilizing a two-fluid model. In this model, erythrocytes are considered a non-Newtonian fluid in the core region, while plasma is treated as a Newtonian fluid in the peripheral layer. The core region's non-Newtonian fluid is assumed to follow both Herschel-Bulkley and Casson models. To solve the resulting non-linear partial differential equations, a perturbation technique is applied. The study yields expressions for various flow parameters under the two-fluid Casson model. The analysis conducted in this paper reviews existing research and concludes that the two-fluid Casson model exhibits significantly lower values for parameters such as plug core radius, pressure drop, wall shear stress and flow resistance compared to the two-fluid Herschel-Bulkley model. Therefore, it is suggested that the two-fluid Casson model is a more valuable approach for studying blood flow in stenosed arteries.

Keywords: *Casson Model, Herschel-Bulkley Model, Peripheral Layer, Plug Core Radius, Stenosed Artery, Womersley*

Introduction

Blood vessels are integral components of the human cardiovascular system responsible for transporting blood throughout the body. There are three primary types of blood vessels: arteries, which convey blood away from the heart; capillaries, facilitating the precise exchange of fluids and chemicals between the blood and tissues; and veins, which transport blood from the capillaries back to the heart. The term vascular, denoting a connection to blood vessels, originates from the Latin word that translates to vessel. It's worth noting that certain structures such as cartilage and the eye's lens lack blood vessels and fall into a distinct category.

Arteries play a crucial role in the cardiovascular system, responsible for supplying oxygen and essential nutrients to all cells, as well as eliminating carbon dioxide and waste products. They also help maintain the body's optimal pH balance and facilitate the circulation of proteins and

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immune cells. In developed nations, the top two causes of mortality, namely heart attacks (myocardial infarctions) and strokes, can often be traced back to a gradual and progressive deterioration of the arterial system over the years. This decline in arterial health contributes significantly to these life-threatening conditions.

Narrowing refers to the abnormal constriction of a vessel or another hollow organ or structure and is also commonly referred to as a stricture. The term stricture is typically employed when the narrowing is a result of the smooth muscle contracting, as seen in conditions like achalasia or Prinz metal angina. On the other hand, stenosis is a term often used when the narrowing is due to a lesion that reduces the lumen's space, such as in the case of atherosclerosis. Atherosclerosis is a medical condition characterized by the buildup of plaque, which consists of fats, cholesterol, calcium, and other substances, on the inner walls of arteries. This plaque accumulation can lead to the narrowing and hardening of the arteries, ultimately restricting blood flow and increasing the risk of various cardiovascular problems, such as heart disease, stroke, and peripheral artery disease. Atherosclerosis is a chronic and progressive condition that can develop over many years and is often associated with risk factors like high cholesterol, high blood pressure, smoking, and diabetes.

Numerous pieces of evidence suggest that vascular fluid dynamics play a pivotal role in the initiation and advancement of arterial stenosis. The narrowing of arteries results from the formation of atherosclerotic plaques that extend into the vessel's lumen, causing arterial stenosis. When an obstruction occurs within an artery, one of the most severe consequences is the increased resistance and subsequent reduction in blood flow to the specific tissue supplied by that artery. Therefore, the presence of stenosis leads to severe disruptions in blood circulation.

Numerous theoretical and experimental endeavours have been undertaken to investigate blood flow characteristics in the presence of arterial pathology. While the assumption of Newtonian behaviour is applicable for high shear rates in larger arteries, blood, being a mixture of cells in plasma, exhibits non-Newtonian behaviour at low shear rates, particularly in small-diameter arteries affected by disease. In diseased conditions, the flow pattern becomes notably womersley. Various researchers have explored the non-Newtonian behaviour and womersley flow of blood through stenosed arteries. Through experimentation, some scientists have demonstrated that in the case of blood flowing through narrow blood vessels, there exists a peripheral layer of plasma surrounding a core region containing a suspension of all erythrocytes. Therefore, for an accurate description of blood flow in such scenarios, it is appropriate to consider blood as a two-fluid model, with erythrocyte suspension in the core region behaving as a non-Newtonian fluid and the plasma in the peripheral region following Newtonian behaviour.

Numerous researchers have analysed and determined that both the Casson fluid model and the Herschel-Bulkley fluid model, both of which account for non-zero yield stress, are more suitable for studying blood flow in stenosed arteries. Casson fluid model is preferred for blood flow investigations due to the simplicity of its mathematical equation. In contrast, the Herschel-Bulkley fluid model is considered less user-friendly because its empirical equation is more complex, involving an additional parameter compared to the Casson model. The parameters relevant to the Casson fluid model, such as viscosity, yield stress and power law, effectively describe the simple shear behaviour of blood. The Casson fluid model is satisfactory for blood flow in tubes with diameters ranging from 130 to 1300, while the Herschel-Bulkley fluid model can be applied to

ISSN: 3048-8036 (Online)
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tubes with diameters of 20 to 100 (Sankar, D. S. & Ismail; 2009, 2010).

Review of Literature

Srivastava and Srivastava (2014) explored the fluid mechanics of an axisymmetric stenosis within an artery, focusing on the impact of wall permeability and assuming a two-fluid model for the flowing blood. They derived expressions for various aspects of blood flow, including its characteristics, impedance, wall shear stress distribution in the stenotic area and shearing stress at the stenosis throat. The study involved conducting numerical computations and presenting the results graphically. The findings were then discussed in a comparative manner to assess the validity and practicality of their model, particularly with regard to the influence of permeability and the peripheral layer on these blood flow characteristics. Biswas and Chakraborty (2010) conducted a study on the pulsatile blood flow within an artery containing a mild stenosis. They approached the modeling of blood flow by considering blood as a two-fluid system, erythrocytes suspended within the core region represented as a Bingham Plastic and the peripheral plasma as a Newtonian fluid. The objective was to examine the effects of body acceleration, non-Newtonian properties of blood and a velocity slip at the wall on blood flow in stenosed arteries. Using perturbation analysis, they derived analytical expressions for various parameters such as velocity profile, Plug-core radius, flow rate, wall shear stress and effective viscosity. The study presented graphical representations of how these flow variables change concerning different parameters, facilitating detailed discussions. Their observations noted an increase in velocity and flow rate, while effective viscosity decreased due to wall slip. Moreover, the influence of body acceleration further enhanced flow rates and speeds. Mishra et al. (2011) focused their study on analyzing the impact of wall permeability in arteries with a composite stenosis. They mathematically derived expressions for various aspects of blood flow, such as flow characteristics, flow resistance, wall shear stress and shearing stress at the stenosis throat. Their findings revealed that impedance increased in proportion to the Darcy number and slip parameter. They observed that flow resistance increased in relation to the size of the stenosis (both height and length) for any specific combination of parameters. Moreover, they concluded that in arteries with permeable stenosis, the impedance exhibited significantly higher values compared to a normal artery without stenosis.

Akbar (2014) studied on analyzing blood flow using the Prandtl fluid model within tapered stenosed arteries. The study presented the governing equations for this model in cylindrical coordinates. Perturbation solutions were developed specifically for velocity, impedance resistance, wall shear stress and shearing stress at the stenosis throat. The main focus was directed towards examining the embedded parameters in scenarios involving converging, diverging and non-tapered conditions. The article concluded with the plotting of streamlines for the arteries under consideration. The observations noted that an increase in Prandtl fluid parameters, stenosis shape and maximum height of the stenosis led to a decrease in the velocity profile. Sankar et al. (2014) conducted a mathematical analysis on the pulsatile blood flow within narrow arteries containing multiple stenoses under body acceleration. They examined the behaviour of blood flow considering two distinct models: a single-phase Herschel-Bulkley fluid model and a two-phase Herschel-Bulkley fluid model. By utilizing the expressions for various flow characteristics from these models, they evaluated the differences in novel flow geometry. Their findings indicated slightly lower values for plug core radius, wall shear stress and longitudinal impedance to flow in

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the two-phase HB fluid model compared to the single-phase H-B fluid model. Kumar et al. (2020) gave an idea on comparative study of non- Newtonian physiological blood flow through elastic stenotic artery with rigid body stenotic artery. Again Kumar et al. (2022) worked on a two- layered model of blood flow for stenosed artery along with the peripheral layer. Rakshit et al. (2023) studied on a mathematical model of flow of blood in a segment of an artery by a non-homogeneous approach. The study observed a notable decrease in velocity with increased yield stress of the fluid, while noting the opposite behaviour for longitudinal impedance to flow. Furthermore, they observed that velocity distribution and flow rate were higher in the two-phase Herschel-Bulkley fluid model than in the single-phase Herschel-Bulkley fluid model. Additionally, they concluded that mean velocity estimates increased with higher body acceleration, but this trend was reversed with an increase in stenosis depth.

Formulation of Problem

Consider the axisymmetric flow of blood in a two-layered structure passing through an axisymmetric constriction. The geometry of the stenosis, which are presumed to exist within the arterial wall segment, are as follows:

$$\frac{R(z), R(z)}{R_0} = (1, \beta) - \frac{(\delta \delta_1)}{2R_0} R_0 \left\{ 1 + \cos \frac{2\pi}{L_0} \left(z - d - \frac{L_0}{2} \right) \right\} \;;\; d \le z \le d + L_0 \tag{1}$$

$$= (1, \beta) \qquad ; \quad otherwise$$

where z represents the axial coordinate and $R(z)$ and $R_1(z)$ correspond to the tube radius and the interface with the constriction, respectively. R_0 stands for the artery's radius in the absence of stenosis, L_0 denotes the stenosis length, L signifies the tube length, and d pinpoints the stenosis location. The parameter β represents the ratio of the central core radius to the tube radius in the unobstructed region, and δ and δ_1 pertain to the maximum height of the stenosis and the bulging of the interface, respectively.

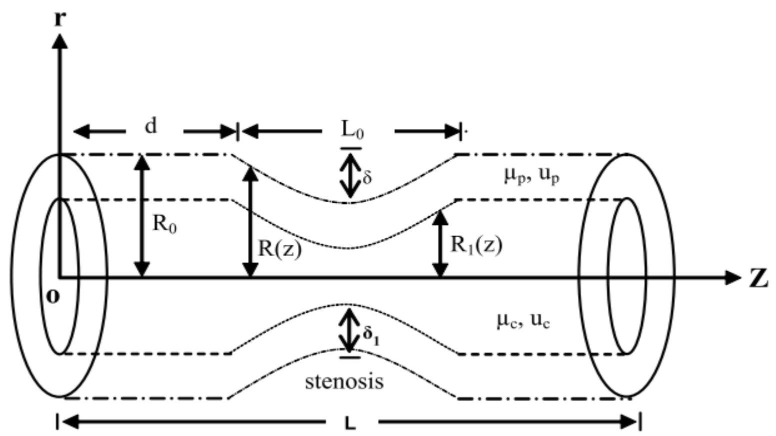

Figure 1: Two-phase blood flow through an axisymmetric stenosis with permeable wall.

In this scenario, the blood flow is assumed to be modeled as a two-layered Newtonian fluid. The governing equations that describe the laminar, steady, one-dimensional flow, particularly in the case of a mild stenosis where δ is significantly smaller than R_0, can be expressed as follows:

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$$\frac{dp}{dz}=\frac{\mu_p}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u_p \quad ; \; R_1(z)\le z\le R(z) \tag{2}$$

$$\frac{dp}{dz}=\frac{\mu_c}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u_c \quad ; \; 0\le z\le R_1(z) \tag{3}$$

where $R_1(z)$ is the radius of the central layer, (μ_p, u_p) and (μ_c, u_c) are(viscosity, velocity) of fluid in the peripheral layer $(R_1(z)\le z\le R(z))$ and central layer $(0\le z\le R_1(z))$ respectively, p represents pressure, while coordinates (r,z) are (radial, axial) coordinates in the two-dimensional cylindrical polar coordinate system.The prescribed boundary conditions are as follows:

$$\frac{\partial u_c}{\partial r}=0, \quad \text{at} \quad r=0 \tag{4}$$

$$u_p=u_c \text{ and } \quad \mu_p\frac{\partial u_p}{\partial r}=\mu_c\frac{\partial u_c}{\partial r} \quad \text{at} \quad r=R_1(z) \tag{5}$$

$$u_p=u_B \text{ and } \quad \frac{\partial u_p}{\partial r}=\frac{\alpha}{\sqrt{k}}\left(u_B-u_{porous}\right) \quad \text{at} \quad r=R(z) \tag{6}$$

where, $u_{porous}=-\frac{k}{\mu_p}\frac{dp}{dz}$ = flow velocity in the permeable boundary, u_B is the slip velocity, μ_p is plasma viscosity in the peripheral layer, k is Darcy number, α is slip parameter.

Mathematical Formulation

Solving the differential equations (2), (3) with the specified boundary conditions (4), (5) and (6) provides us with the velocity $(u_p \text{ and } u_c)$ expressions as: follows:

$$u_p=-\frac{R_0^2}{4\mu_p}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^2-\left(\frac{r}{R_0}\right)^2-2\left(\frac{R}{R_0}\right)\left(\frac{\sqrt{k}}{\alpha R_0}\right)+\frac{k}{R_0^2}\right\} \tag{7}$$

$$u_c=-\frac{R_0^2}{4\mu_p}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^2-\mu\left(\frac{r}{R_0}\right)^2-(1-\mu)\left(\frac{R_1}{R_0}\right)^2-2\left(\frac{R}{R_0}\right)\left(\frac{\sqrt{k}}{\alpha R_0}\right)+4\frac{k}{R_0^2}\right\} \tag{8}$$

with $\mu=\frac{\mu_p}{\mu_c}$

The Volumetric flow rate Q can be expressed as:

$$Q=2\pi\left\{\int_0^{R_1} r\,u_c\,dr+\int_{R_1}^{R} r\,u_p\,dr\right\}$$

$$Q=-\frac{\pi R_0^4}{8\mu_p}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^4-(1-\mu)\left(\frac{R_1}{R_0}\right)^2+\frac{8k}{R_0^2}\left(\frac{R}{R_0}\right)^2-\frac{4\sqrt{k}}{\alpha R_0}+4\left(\frac{R}{R_0}\right)^3\right\} \tag{9}$$

Following the argument [Shukla et.al (1980) and Srivastava (2010)] that the total flux is equal to the sum of the fluxes across the two regions (peripheral and central), one derives the relations: $R_1=\beta R$ and $\delta_1=\beta\delta\,(0\le\beta\le 1)$. An application of these relations into the equation (9), yields

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$$\frac{dp}{dz} = -\frac{8\mu_p Q}{\pi R_0^{\,4}}\phi(z), \tag{10}$$

where $\phi(z) = \dfrac{1}{\left\{\left[1-(1-\mu)\beta^4\right]\left(\dfrac{R}{R_0}\right)^4 + \dfrac{8k\left(\dfrac{R}{R_0}\right)^2}{R_0^{\,2}} - 4\sqrt{k}\,\dfrac{\left(\dfrac{R}{R_0}\right)^3}{\alpha R_0}\right\}}$

The pressure drop $\Delta p = p \;\; at \;\; z=0$ and $\Delta p = -p \;\; at \;\; z = L$ across the stenosis in the tube of length, L is obtained as:

$$\Delta p = \int_0^L \left(-\frac{dp}{dz}\right) dz$$

$$\Delta p = \frac{8\mu_p Q}{\pi R_0^{\,4}}\left\{\int_0^d \left[\phi(z)\right]_{R/R_0=1} dz + \int_d^{d+L_0} \phi(z)\,dz + \int_{d+L_0}^{L} \left[\phi(z)\right]_{R/R_0=1} dz\right\} \tag{11}$$

The analytical evaluation of the second integral on the right-hand side of equation (11) is a formidable task and therefore shall be evaluated numerically, whereas the evaluation of first and third integrals are straight forward. Using now the definitions from the published literature Srivastava et.al (2010) and Young (1968), one derives the expressions for the impedance (flow resistance), λ, the wall shear stress distribution in stenotic region, τ_w and the shear stress at the stenosis throat, τ_s in their non-dimensional form as:

$$\lambda = \mu\left\{\frac{\eta_1\left(1-\dfrac{L_0}{L}\right)}{\eta_1} + \left(\frac{\eta_1}{2\pi}\right)\left(\frac{L_0}{L}\right)\int_0^{2\pi}\psi(\theta)\,d\theta\right\} \tag{12}$$

$$\tau_w = \frac{\mu\eta_1}{\left\{\left[1-(1-\mu)\beta^4\right]\left(\dfrac{R}{R_0}\right)^3 + \dfrac{8k\left(\dfrac{R}{R_0}\right)}{R_0^{\,2}} - 4\sqrt{k}\,\dfrac{\left(\dfrac{R}{R_0}\right)^2}{\alpha R_0}\right\}} \tag{13}$$

$$\tau_s = \left[\tau_w\right]_{\frac{R}{R_0}=1-\frac{\delta}{R_0}} \tag{14}$$

where $\psi(\theta) = \left[\phi(z)\right]_{\frac{R}{R_0}=a+b\cos\theta}$, $a = 1-\dfrac{\delta}{2R_0}$, $b = \dfrac{\delta}{2R_0}$

$$\eta_1 = 1 + \frac{8k}{R_0^{\,2}} - \frac{4\sqrt{k}}{\alpha R_0}$$

$$\eta = 1-(1-\mu)\beta^4 + \frac{8k}{R_0^{\,2}} - \frac{4\sqrt{k}}{\alpha R_0}$$

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$\lambda = \dfrac{\bar{\lambda}}{\lambda_0}$, $(\tau_w, \tau_s) = \dfrac{(\bar{\tau}_w, \bar{\tau}_s)}{\tau_0}$, $\lambda_0 = \dfrac{8\mu_c L}{\eta_1 \pi R_0^4}$ and $\tau_0 = \dfrac{4\mu_c Q}{\eta_1 \pi R_0^3}$ are the flow resistance and shear stress, respectively for a single-layered Newtonian fluid in a normal artery (no stenosis) with permeable wall and

$(\bar{\lambda}, \bar{\tau}_w, \bar{\tau}_s)$ are respectively, (the impedance, the wall shear stress and the shearing stress at stenosis throat) obtained from the definitions Young (1968).

$$\bar{\lambda} = \frac{\Delta p}{Q},\ \bar{\tau}_w = -\left(\frac{R}{2}\right)\left(\frac{dp}{dz}\right),\ \bar{\tau}_s = \left[\bar{\tau}_w\right]_{\frac{R}{R_0}=1-\frac{\delta}{R_0}}$$

Result and Discussion

In this investigation, the results of the study quantitatively, computer codes are developed to evaluate the analytical result for flow resistance, λ, the wall shear stress, τ_w, and shear stress at the stenosis throat, τ_s obtained above in equations (12) to (14) for various parameter values and some of the critical results are displayed graphically in figures (2) to (13). The various parameters are selected from Beavers and Joseph (1967), Srivastava et.al (2012), and Young (1968) as:

$L_0(cm) = 1$; $L(cm) = 1, 2, 5, 10$; $\alpha = 0.1, 0.2, 0.3, 0.5$; $\sqrt{k} = 0, 0.1, 0.2, 0.3, 0.4, 0.5$;

$\beta = 1, 0.95, 0.90$; $\mu = 1, 0.5, 0.3, 0.1$; and $\delta/R_0 = 0, 0.5, 0.10, 0.45, 0.20$; etc.

It is worth mentioning here that present study corresponds to impermeable artery case, to single layered model study, and no stenosis case for parameter values $\sqrt{k} = 0$ (here and after called Darcy number); $\beta = 1$ or $\mu = 1$, and $\delta/R_0 = 0$; respectively.

The flow resistance λ, increases with the stenosis height, δ/R_0, for any given set of parameters. At any given stenosis height, δ/R_0, λ decreases with the peripheral layer viscosity, μ from its maximal magnitude obtained in a single-layered study (i.e., $\mu = 1$ or $\beta = 1$; Fig. 2).

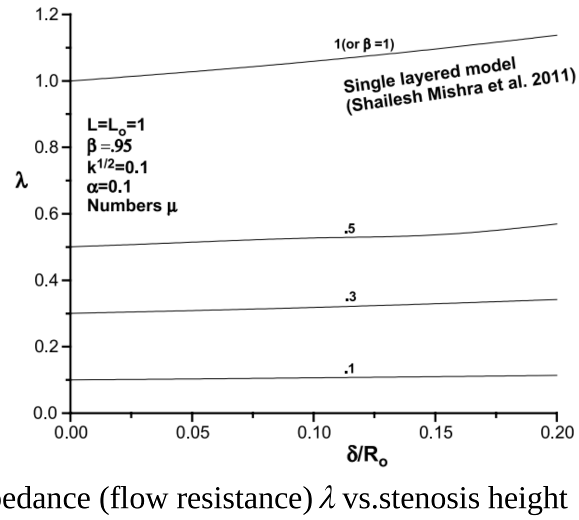

Figure 2: Impedance (flow resistance) λ vs.stenosis height δ/R_0 for different μ.

One observes that at any given stenosis height, δ/R_0, the impedance, λ increases with the slip parameter, α (Fig. 3). The blood flow characteristic, λ increases with the Darcy number, $\sqrt{k}$ at

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any given stenosis height, δ/R_0 (Fig. 4). The impedance, λ decreases with increasing tube length L which inturns implies that λ, increases with increasing value of L (stenosis length, Fig. 5).

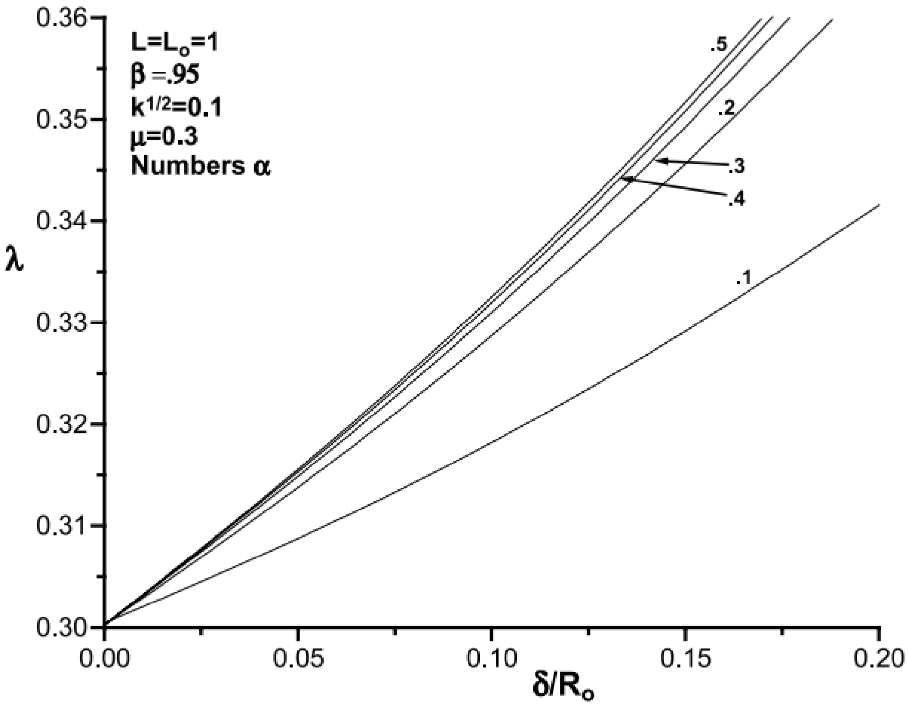

Figure 3: Impedance (flow resistance) λ vs.stenosis height δ/R_0 for different α .

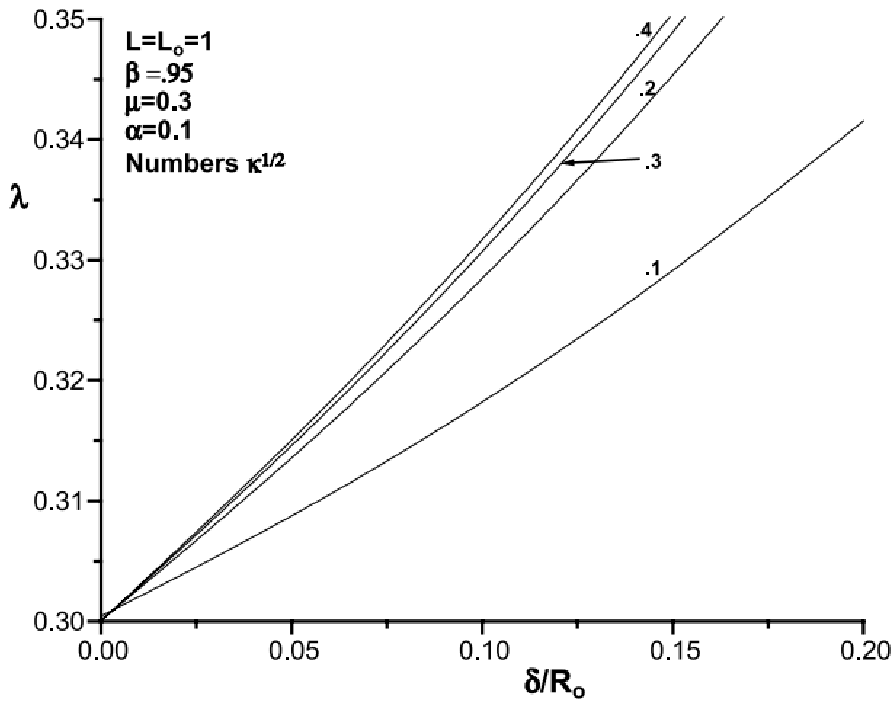

Figure 4: Impedance (flow resistance) λ vs.stenosis height δ/R_0 for different $k^{1/2}$.

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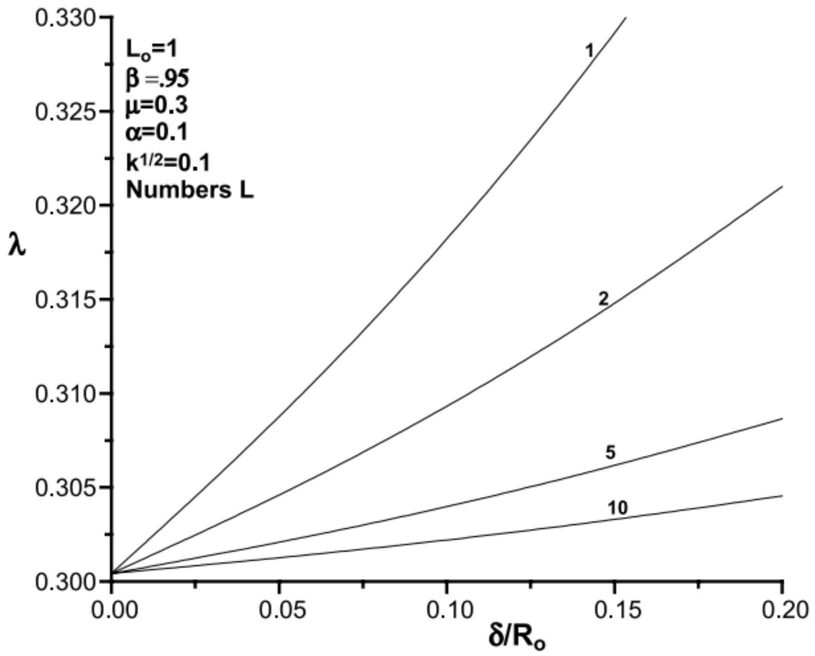

Figure 5: Impedance (flow resistance) λ vs.stenosis height δ/R_0 for different L.

One observes that the flow resistance, λ decreases rapidly with increasing value of the Darcy number, $\sqrt{k}$ from its maximal magnitude at $\sqrt{k}=0$ (impermeable wall) in the range $0 \le \sqrt{k} \le 0.15$ and afterwards assumes an asymptotic value with increasing values of the Darcy number, $\sqrt{k}$ (Fig. 6). We notice that the blood flow characteristic, λ increases with the slip parameter, α from its minimal magnitude at $\alpha = 0.1$ and approaches to an asymptotic magnitude when α increases from 0.2 (Fig. 7).

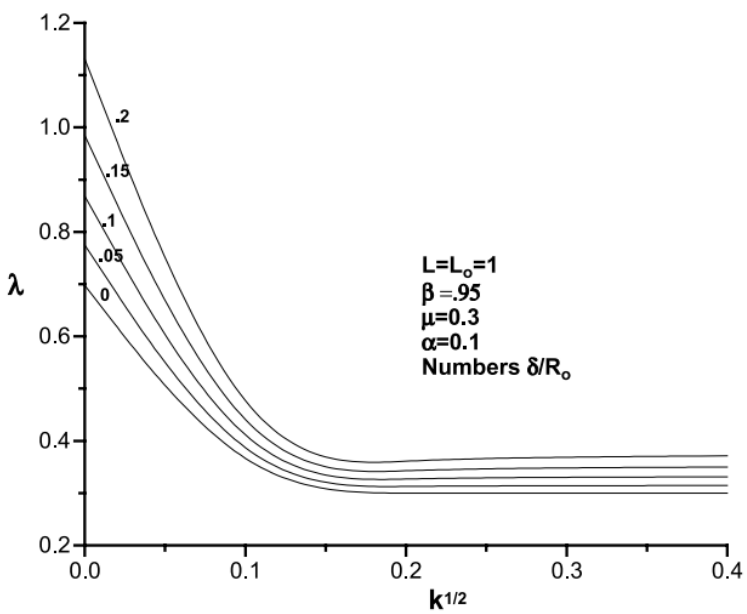

Figure 6: Impedance (flow resistance) λ versus Darcy number $k^{1/2}$ for different stenosis height

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δ/R_0 .

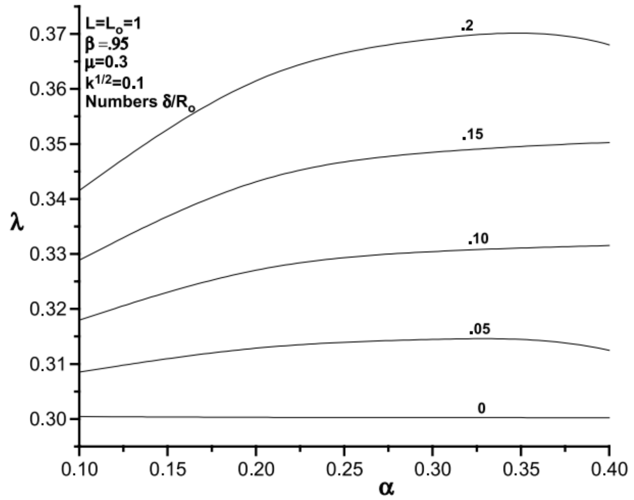

Figure 7: Impedance (flow resistance) λ versus slip parameter α for different stenosis height δ/R_0 .

The wall shear in the stenotic region, τ_w increases from its approached value at z/L_0 to its peak value at $z/L_0 = 0.5$ and then decreases from its peak value to its approached value at the end point of the constriction profile at $z/L_0 = 1$ for any given set of parameters (Figs. 8–11). The blood flow characteristic, τ_w decreases with the peripheral layer viscosity, μ at any axial location of the constriction profile (Fig. 8).

For any given set of parameters, the wall shear stress, τ_w increases with the stenosis height, δ/R_0 (Fig. 12). The blood flow characteristic, τ_s increases with the slip parameter, α (Fig. 13) for any given set of other parameters. Numerical results reveal that the variations of the shear stress, τs are similar to that of the impedance (flow resistances), λ with respect to any parameter.

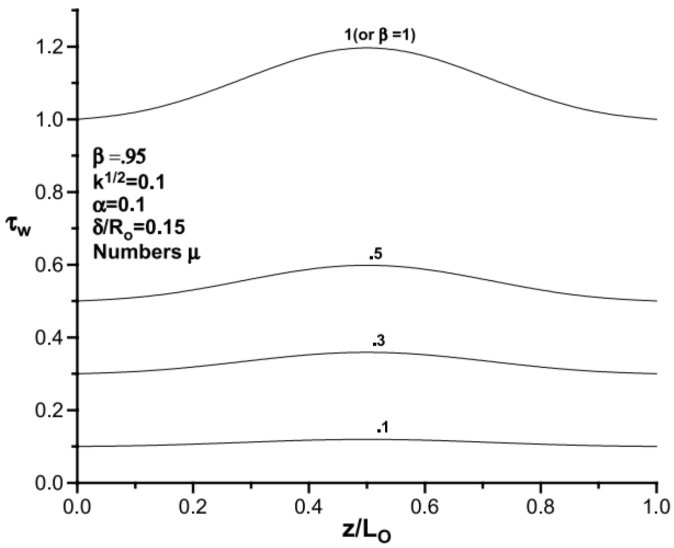

Figure 8: Wall shear stress in the stenotic region for different μ .

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At any point of stenotic region, the wall shear stress, τ_w increases with Darcy number, $\sqrt{k}$ (Fig. 9). The flow characteristic τ_w also increases with the slip parameter, α at any axial location in the stenotic region (Fig. 10).

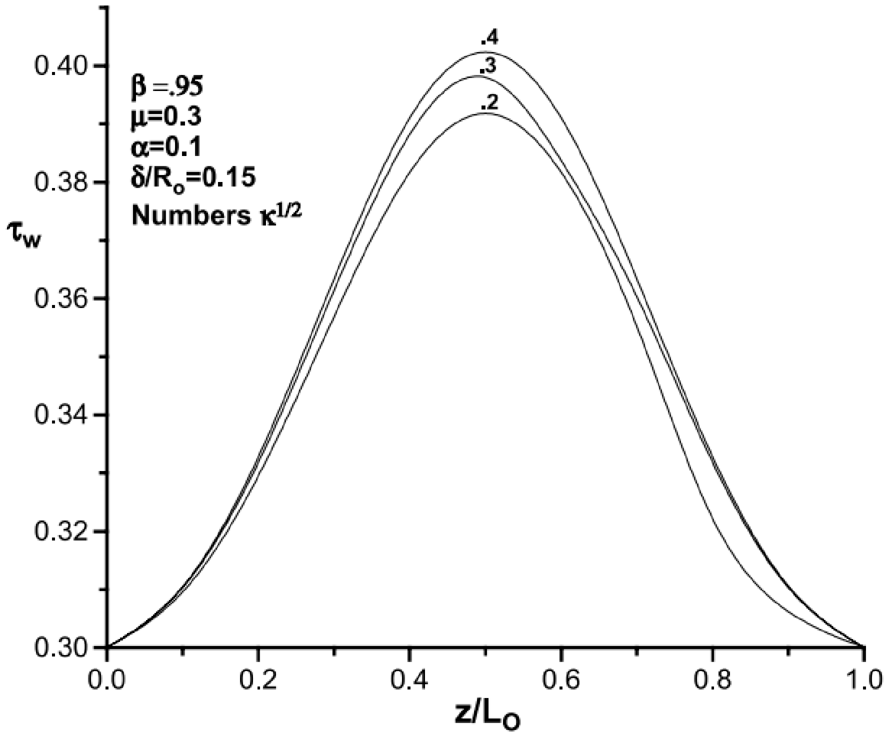

Figure 9: Wall shear stress in the stenotic region for different $k^{1/2}$.

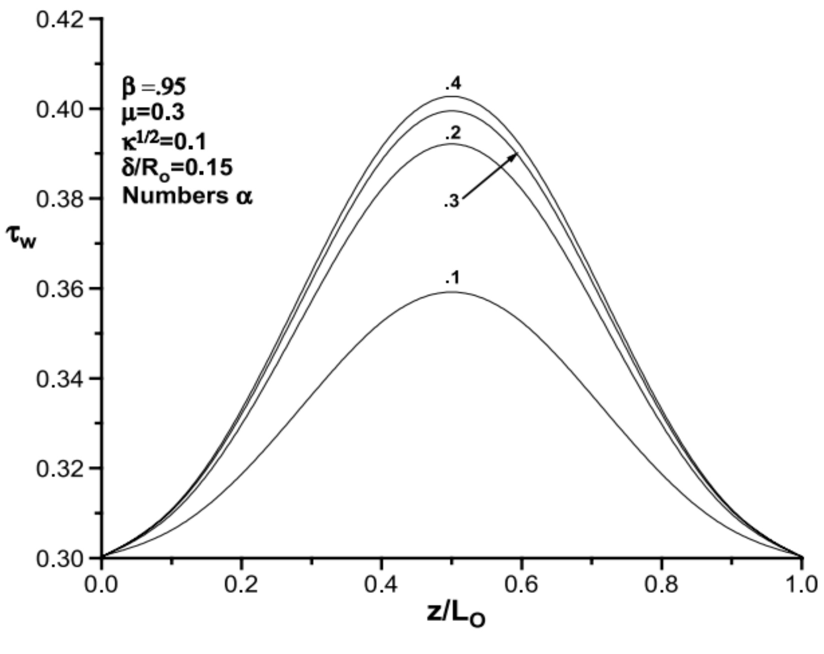

Figure 10: Wall shear stress in the stenotic region for different α.

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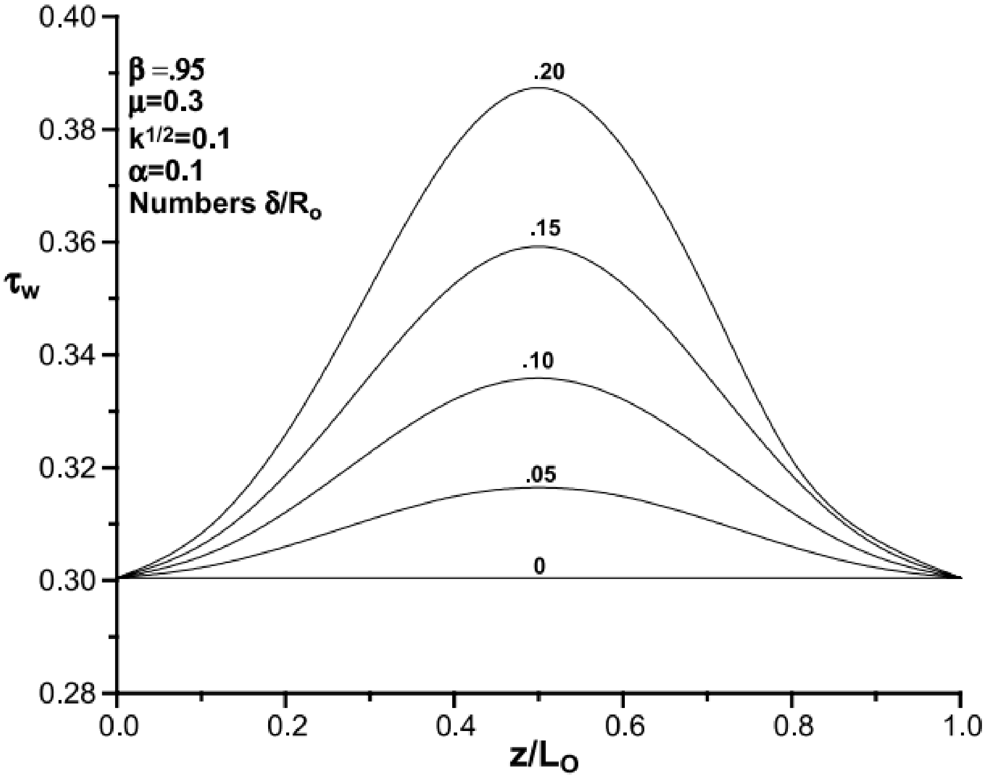

Figure 11: Wall shear stress in the stenotic region for different δ/R_0 .

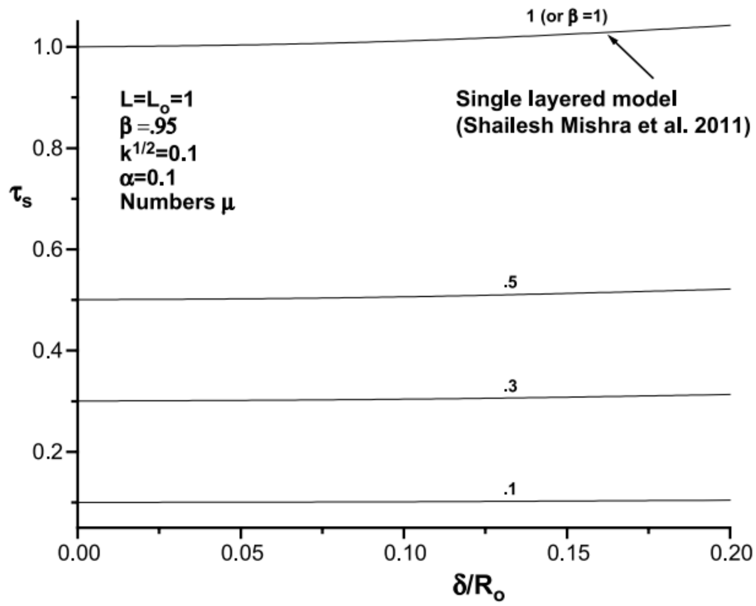

Figure 12: Shear stress at the stenosis throat, τ_s versus stenosis height δ/R_0 , for different μ .

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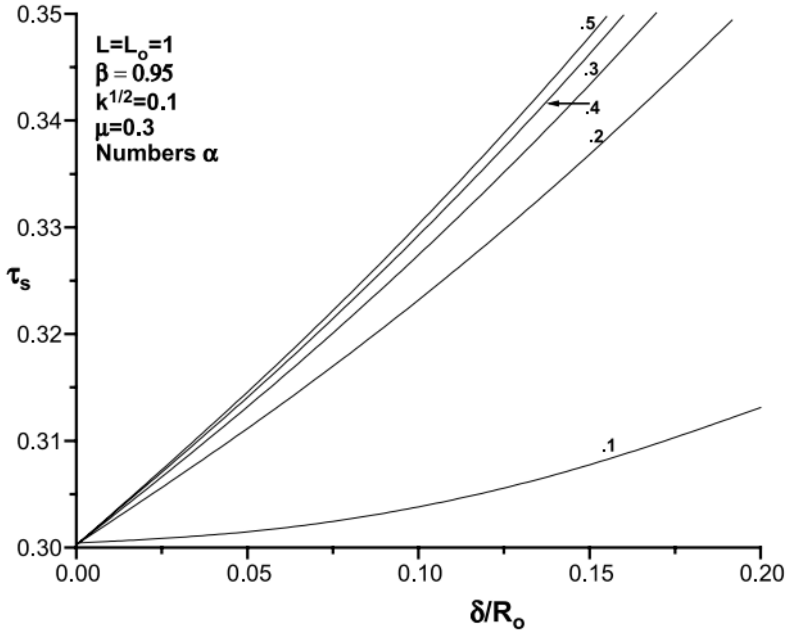

Figure 13: Shear stress at the stenosis throat, τ_s versus stenosis height δ/R_0 , for different μ .

Conclusions

This research focused on the impact of artery wall porosity and the surrounding peripheral layer on blood flow characteristics in the presence of a pathology, specifically, a two-fluid blood flow model of Newtonian fluid through an axisymmetric arterial pathology with a leaky wall. This investigation aimed to simultaneously observe how the wall porosity and the peripheral layer influence blood flow characteristics within this pathological context. The findings suggest that, for any given parameter set, characteristics such as impedance and wall shear stress are of lower magnitude in the two-fluid model compared to the corresponding values obtained in a one-fluid analysis. The electrical resistance decreases as the Darcy number increases, reaching its peak at zero Darcy number, signifying the impact of the wall's impermeability. Consequently, it's concluded that the presence of porosity in the artery wall and the existence of the peripheral layer aid the functionality of the pathological artery. Moreover, it was observed that factors such as pressure drop, plug core radius, wall shear stress and flow resistance exhibit significantly lower values in the two-fluid Casson model compared to those of the two-fluid Herschel-Bulkley model. This suggests that the two-fluid Casson model is more advantageous for studying blood flow through constricted arteries than the two-fluid Herschel-Bulkley model.

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ISSN: 3048-8036 (Online)
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Indian Journal of Multidisciplinary Research (IJOMR)
(Yearly, Blind Peer-Reviewed, Multidisciplinary Journal)
Home Page: https://ijomr.org
ISSN: 3048-8036 (Online)
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