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Mathematical Modeling of Blood Flow within the Mildly Stenosed Artery with the Oscillatory MHD Effect

Goutam Das

Assistant Professor, Dept. of B.Ed., Madhyamgram B.Ed. College, Kolkata, WB, India

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Abstract:
The aim of this research work is to investigate the impact of oscillating MHD blood flow in stenotic arteries. Assuming a Newtonian fluid, the analytical and numerical findings for oscillatory MHD blood flow are produced. Also, it was assumed that, in comparison to the radius of an open-ended tube, the maximum height of surface roughness had a cosine form and was inconsequential. This study focuses into the fluid mechanics of blood flow in stenosed arteries using a mathematical model. The numerical solutions were presented for the effect of the magnetic field on the instantaneous flow rate, which decreases as the Hartman number increases.

Keywords: *Newtonian fluid, Oscillatory MHD blood flow, Cosine form, Stenosed arteries, Hartman number.*

Introduction

In the field of cardiovascular health, it is of utmost significance to comprehend the intricate dynamics of blood flow within arteries, especially those affected by mild stenosis. This understanding is essential for accurate diagnosis and effective treatment of a wide array of cardiovascular ailments. The presence of stenosis, a narrowing of the arterial lumen, significantly alters blood flow patterns, potentially leading to adverse health outcomes such as thrombosis or tissue ischemia. Furthermore, incorporating the oscillatory Magnetohydrodynamic (MHD) effect into the mathematical modeling of blood flow adds another layer of complexity, as it accounts for the interaction between the flowing blood and an external magnetic field, a phenomenon particularly relevant in biomedical engineering applications. Therefore, in this study, we embark on a journey to explore the intricacies of blood flow within mildly stenosed arteries while considering the influence of the oscillatory MHD effect, aiming to unravel insights crucial for both clinical understanding and technological advancements in cardiovascular medicine.

Several investigations have been conducted on blood flow in stenosed arteries, but few of them have looked at oscillatory MHD flow and have never used a mathematical model. An attempt is made to formulate an analysis for such a problem in the present study. Jain et al. (2010) investigated blood flow in a stenosed artery under the MHD effect within the porous medium using mathematical modeling. Analytical expressions have been obtained for share stress at the wall, pressure gradient, axial velocity, volumetric flow rate, and resistance to blood flow. They found that the flow patterns are considerably controlled by the magnetic field. Additionally, they discovered that a variety of factors, mainly the porosity constant and magnetic number, had an impact on the blood flow within the stenosed artery. Blood flow inside a multistenosed artery under the influence of an externally applied magnetic field was investigated by Bali and Awasthi (2012). They considered an artery to be a round tube. By simulating blood as a Casson fluid, the impact of the non-Newtonian character of blood in small blood vessels has been considered. The study presents a graphic representation of the effects of many parameters on the velocity field, including the height of stenosis, shear stress on the wall at the stenotic surface and volumetric flow rate in the stenotic region. Using heat transfer and a bifurcated artery with minor stenosis in the parent lumen, Srinivasacharya and Rao (2015) investigated the impact of MHD on the couple stress fluid flow and provided numerical solutions for steady MHD blood flow. They assumed that blood was the couple stress fluid. The irregular boundary is transformed into a clearly defined boundary by coordinate transformation, which is based on the non-dimensionalization of the governing equations.

The finite difference method is used to numerically solve the resulting system of equations. A graphical representation is provided of the change in shear stress, flow rate, and impedance in the immediate region of the flow divider, along with the related physical data. The effect of varying viscosity on MHD-inclined arterial blood flow with a chemical reaction was studied by Tripathi and Sharma (2018). It is thought that the blood's variable viscosity varies with the hematocrit ratio. They used an analytical scheme and the homotopy perturbation method to solve the governing non-linear differential equations and find a solution for the blood flow's velocity, temperature, and concentration profiles. They found that in an incline artery, shear stress on the wall at the stenosis throat increases with applied magnetic field values, while it decreases with increasing chemical reaction and porosity parameter values. The influence of viscous dissipation and chemical reactions on MHD oscillatory blood flow in a tapered asymmetric channel was studied by Sasikumar and Senthamarai (2022). They treated blood as an optically thick, viscoelastic fluid passing through a porous material and magnetic force that is thought to travel normally throughout the neurological system. An analysis is done on the impact of chemical reactions and viscous dissipation on blood flow.

Mathematical Formulation:

In this current analysis, we view the artery as a circular, rigid, and cylindrical tube. Using the coordinate system (r, z, t), where the z-axis aligns with the artery's axis and the r-axis corresponds to its radius, we examine a laminar flow of blood, presumed to adhere to Newtonian characteristics, within an artery affected by mild stenosis. Throughout this study, the blood has maintained constant density and viscosity. The cylindrical shape of stenosis within the arterial segment is given by:

$$\frac{R(z)}{R_0} = \frac{\epsilon}{2R_0}\left(1 + \cos\frac{\pi z}{d}\right) \quad (1)$$

where the radius of the stenosed arterial region is denoted by $R(z)$, the radius of the normal artery is represented by R_0, the semi-length of the stenosis is denoted by d and the maximum height of the stenosis is represented by ϵ, such that $\frac{\epsilon}{R_0} << \epsilon$ (Figure 1).

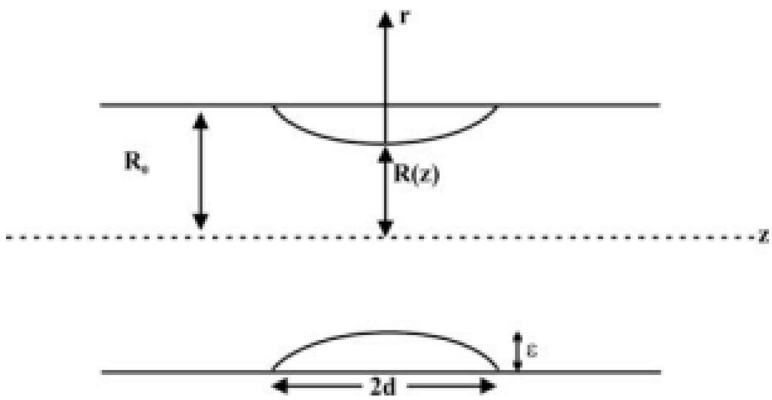

Figure 1: *Cylindrical Flow Geometry of Stenosed Artery*

This mathematical model, in which it is also introduced as a magnetic field, can be analyzed using the following governing equation:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{v}{R_0^2}\left\{\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right\} + \frac{v}{R_0^2}\beta R_0^2 \frac{\sigma}{\mu} u$$

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \lambda\left\{\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right\} + \lambda M^2 u \tag{2}$$

Where p is the fluid pressure, v is the kinematic viscosity, ρ is the fluid density, μ is the viscosity of the fluid, and u is the fluid velocity in the axial direction,

$$\lambda = \frac{v}{R_0^2},\ \beta^2 = \frac{\rho R_0^2 \omega}{\mu} \text{ and } M^2 = \beta R_0^2 \frac{\sigma}{\mu}.$$

While the following are the boundary conditions:

$$\left.\begin{array}{ll} u = 0 & \text{on } r = \frac{R}{R_0} \\ \frac{\partial u}{\partial r} = 0 & \text{on } r = 0 \end{array}\right\} \tag{3}$$

In order to solve the problem, let's now consider the following expression:

$$u(r,t) = \bar{u}(r)e^{i\omega t}, \qquad -\frac{\partial p}{\partial z} = Pe^{i\omega t} \tag{4}$$

$$\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr} - i\left(\frac{\rho R_0^2 \omega}{\mu} + i\beta R_0^2 \frac{\sigma}{\mu}\right)\bar{u} = -\frac{R_0^2}{\mu}P \tag{5}$$

We can now write equation (5) as follows:

$$\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr} - i(\beta^2 + iM^2)\bar{u} = -\frac{R_0^2}{\mu}P$$

$$\frac{d^2\bar{u}}{dr^2} + \frac{1}{r}\frac{d\bar{u}}{dr} - i\alpha^2\bar{u} = -\frac{R_0^2}{\mu}P \tag{6}$$

Where $\alpha = \beta^2 + iM^2$

Hence the expression (4), and its corresponding boundary conditions are as follows:

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$$\left.\begin{aligned} \bar{u} &= 0 \qquad \text{at } r = \frac{R}{R_0} \\ \frac{d\bar{u}}{dr} &= 0 \qquad \text{at } r = 0 \end{aligned}\right\} \tag{7}$$

According to the boundary conditions (2), the solution to equation (2.6) is:

$$\bar{u}(r) = \frac{PR_0^2}{i\mu\alpha^2}\left[\frac{J_0\left(\frac{\alpha r}{R_0} i^{\frac{3}{2}}\right)}{J_0\left(\frac{\beta R}{R_0} i^{\frac{3}{2}}\right)}\right] \tag{8}$$

In this case, J_0 represents the complex argument Bessel function of order zero.
The axial velocity can thus be expressed as follows:

$$u(r,t) = \frac{PR_0^2}{i\mu\alpha^2}\left[\frac{J_0\left(\frac{\alpha r}{R_0} i^{\frac{3}{2}}\right)}{J_0\left(\frac{\beta R}{R_0} i^{\frac{3}{2}}\right)}\right] e^{i\omega t} \tag{9}$$

Using the terminology provided by McLachlan (1934)

$$J_0\left(z i^{\frac{3}{2}}\right) = M_0(z) e^{i\theta} 0^{(z)}$$

This might alternatively be written as:

$$u(r,t) = \frac{PR_0^2 M_0}{\mu\alpha^2}\left[\sin(\omega t + \epsilon_0) - i\cos(\omega t + \epsilon_0)\right] \tag{10}$$

$$\text{where } \epsilon_0 = \tan^{-1}\left[\frac{h_0 \sin\phi}{1 - h_0\cos\phi}\right],$$

$$\phi = \theta_0\left(\frac{\alpha R}{R_0}\right) - \theta_0\left(\frac{\alpha r}{R_0}\right),$$

$$M_0 = \left[1 + h_0^2 - 2h_0\cos\phi\right]^{\frac{1}{2}}$$

$$\text{and } h_0 = \frac{M_0\left(\frac{\alpha r}{R_0}\right)}{M_0\left(\frac{\alpha R}{R_0}\right)}$$

Now, if P cos ωt represents the real component of the simple harmonic pressure gradient, the axial velocity formulation is:

$$u(r,t) = \frac{PR_0^2 M_0}{\mu\alpha^2}\sin(\omega t + \epsilon_0) \tag{11}$$

along with the volumetric flow rate:

$$Q = \frac{\pi R_0^4 P}{i\mu\alpha^2}\left(\frac{R}{R_0}\right)\left[\frac{R}{R_0} - \frac{2J_1\left(\frac{\alpha R}{R_0} i^{\frac{3}{2}}\right)}{i^{\frac{3}{2}} J_0\left(\frac{\alpha R}{R_0} i^{\frac{3}{2}}\right)}\right] e^{i\omega t} \tag{12}$$

For pressure gradient P cos ωt, the flow rate is:

$$Q = \frac{nPR_0^4 M_1}{\mu\alpha^2}\left(\frac{R}{R_0}\right)\sin(\omega t + \epsilon_1) \tag{13}$$

Where $\epsilon_1 = \tan^{-1}\left[\frac{h_1 \sin\theta}{\left(\frac{R}{R_0} - h_1 \cos\psi\right)}\right]$,

$$M_1 = \left[\left(\frac{R}{R_0}\right)^2 + h_1^2 - 2\left(\frac{R}{R_0}\right) h_1 \cos\psi\right]^{\frac{1}{2}},$$

$$h_1 = \frac{2M_1\left(\frac{\alpha R}{R_0}\right)}{\alpha M_1\left(\frac{\alpha R}{R_0}\right)},$$

$$\psi = \frac{3\pi}{4} - \theta_1\left(\frac{\alpha R}{R_0}\right) + \theta_0\left(\frac{\alpha R}{R_0}\right)$$

The shear stress at the wall:

$$\tau_R = \mu\left(\frac{\partial u}{\partial r}\right)_{r=R}$$

Results and Discussion:

In order to solve the problem numerically, let us assume that $\frac{2d}{l} = 1$ and $\frac{R}{R_0} = 1 - \frac{\epsilon}{R_0}$. Considering that the frequency parameter α is crucial to the flow pattern, it will now use it to describe the shear stress, and flow rate of the walls.

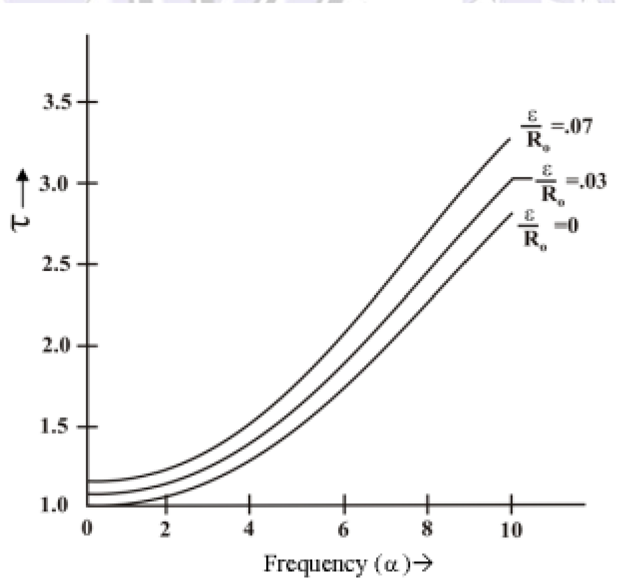

Figure 2: *Variation of shear stress at the wall with frequency parameter α for various stenosis height*

Figure 2 indicates how the shear stress at the wall varies with frequency for various stenosis heights. The study has found that when the stenosis height $\left(\frac{\epsilon}{R_0}\right)$ increases for fixed values of the frequency parameter α, the shear stress at the wall $|\tau|$ also increases. Put another way, shear stress increases as the stenosis height does.

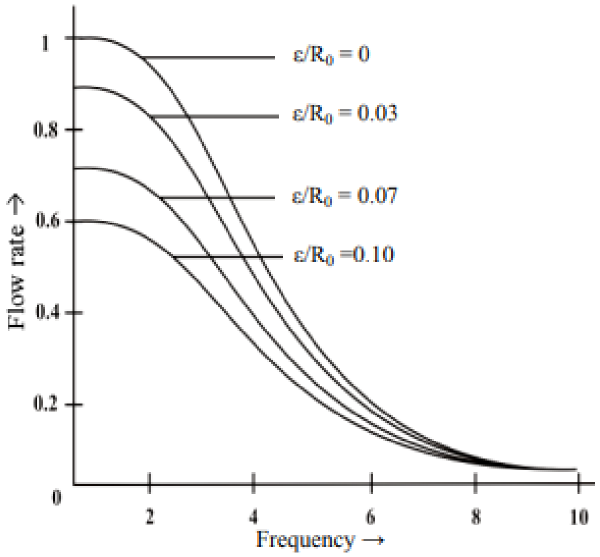

Figure 3: *Variation of instantaneous rate of flow with frequency for various stenosis height*

The instantaneous flow rate varies with frequency for various stenosis heights, as displayed in Figure 3. The flow rate was similarly found to decrease with increasing stenosis height $\left(\frac{\epsilon}{R_0}\right)$ for a specific value of the frequency parameter $|\alpha|$.

In the interval $0 \leq \alpha < 1$, the deviation between any two successive curves is roughly constant; outside of this range, it dramatically falls for any values of $|\alpha|$ that lie on the sharply falling portions of the curve.

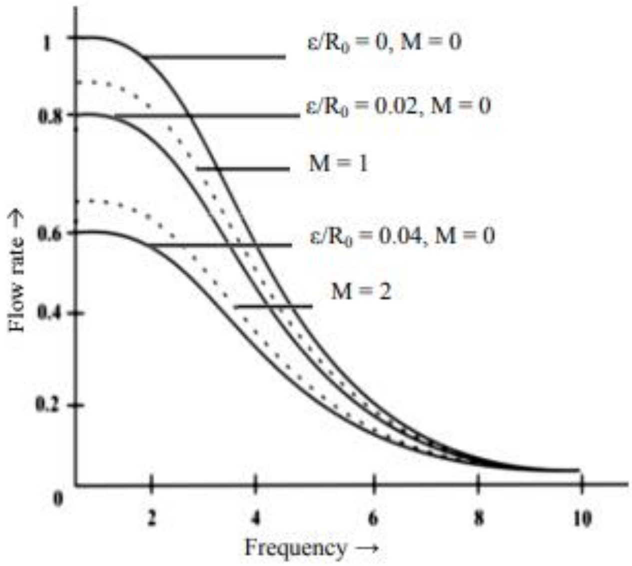

Figure 4: *Variation of instantaneous flow rate with frequency both with and without a Hartmann number*

The instantaneous flow rate variation is displayed in Figure 4 both in the presence and absence of the Hartmann number, or magnetic field. When a magnetic field is present, the increase in Hartmann numbers reduces the variation in instantaneous flow rate, and vice versa. In an instance of no magnetic field, the outcome is the same as that described by Haldar (1987) in the oscillatory flow of blood in a stenosed artery.

Conclusions:

The analytical and numerical results for oscillatory MHD blood flow, which is believed to be a Newtonian fluid, are obtained in order to comprehend the irregular flow conditions of blood in locally constricted blood vessels. It is assumed that the surface roughness in this instance has a cosine form and that its maximum height is extremely slight in relation to the radius of the unconstrained tube. For various values of stenosis height, numerical solutions are given for the instantaneous flow rate, shear stress at the wall, and instantaneous flow rate with frequency, both in the absence and in the presence of Hartmann numbers.

This study suggests the complicated interactions among oscillatory MHD impact, artery stenosis, and blood flow dynamics, offering important new understandings of the intricate behavior of blood flow in mildly stenosed arteries. These results open up new avenues for investigating and improving mathematical models to improve our comprehension of cardiovascular disorders and guide future therapeutic approaches.

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